Local probabilities for random walks conditioned to stay positive

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Let $S_0 = 0, \{S_n,\, n \geq 1\}$ be a random walk generated by a sequence of i.i.d. random variables $X_1, X_2, ...$ and let $\tau^- = \min\{n \geq 1 : S_n \leq 0\}$ and $\tau^+ = \min\{n \geq 1 : S_n > 0\}$. Assuming that the distribution of X_1 belongs to the domain of attraction of an α-stable law we study the asymptotic behavior, as $n \to \infty$, of the local probabilities $\mathbf{P}(\tau^{\pm} = n)$ and prove the Gnedenko and Stone type conditional local limit theorems for the probabilities $\mathbf{P}(S_n \in [x, x + \Delta) | \tau^- > n)$ with fixed Δ and $x = x(n) \in (0, \infty)$.