A Central Limit Theorem for a Randomly Driven Semilinear Parabolic Equation

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Abstract : We study the semilinear parabolic equation

$$u_t = u_{xx} - u^2 + \lambda_\omega(t)\,\delta_0(x), \qquad x \in \mathbb{R},\ t \in \mathbb{R},$$

driven by a source term $\lambda_\omega(t)\delta_0(x)$ at the origin. The intensity of the source is considered to be a positive, stationary and ergodic process. The solution of this equation describes the equilibrium state of a system, in which energy is supplied by the source, and is diffused and dissipated by the Laplacian and the nonlinearity, respectively. We prove that as x tends to infinity, the equilibrium solution $u_\omega(\cdot, x)$ becomes $a.s.$ asymptotic to a steady state solution of the same equation, corresponding to an averaged constant intensity λ_*. Moreover, we study the fluctuations around this steady state solution, under appropriate assumptions on the decay of correlations of the intensity $\lambda_\omega(\cdot)$.